



Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER				CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

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Find the set of values of k for which the system of equations

$$x+5y+6z = 1,$$

 $kx+2y+2z = 2,$
 $-3x+4y+8z = 3,$

has a unique solution and interpret this situation geometrically.	[4]
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2 It is given that

(a)

$x = 1 + \frac{1}{t}$	and	$y = \cos^{-1} t$	for $0 < t < 1$.
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Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{t^2}{\sqrt{1-t^2}}$.	[2]
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(b)

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Show that $\frac{d^2y}{dx^2} = -t^a(1-t^2)^b(2-t^2)$, where <i>a</i> and <i>b</i> are constants to be determined. [4]

$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1 + u^2} \mathrm{d}u.$	[2]

(b) Use the substitution $u = \sinh v$ to show that

$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right).$	[6]

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[6]

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4 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

(a)	Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of A and state the corresponding eigenvalue.	[2]
(b)	Show that the characteristic equation of A is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the ot eigenvalues of A .	ther [3]

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(c)	Use the characteristic equation of

y	r	r	4
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Use the characteristic equation of \mathbf{A} to find \mathbf{A}^{-1} .	[4]
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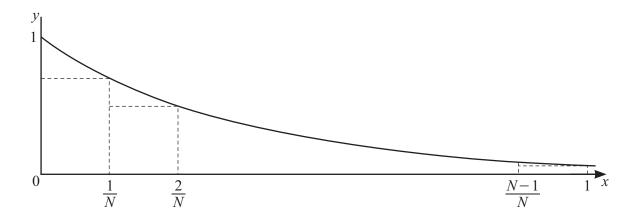
Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{dx}{dt} = -6$.	[10]

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The diagram shows the curve with equation $y = \left(\frac{1}{2}\right)^x$ for $0 \le x \le 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x dx > L_N$, where

$$L_N = \frac{1}{2N(2^{\frac{1}{N}} - 1)}. \tag{4}$$

(b)

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Use a similar method to find, in terms of N , an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x dx$. [4]	.]
	•
Find the least value of N such that $U_N - L_N \le 10^{-3}$. [2]	· !]
	•
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(c)



(d)	Given that $\int_0^1 \left(\frac{1}{2}\right)^x dx = \frac{1}{2 \ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$.

7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2 + 16}$.	[4]



(b) Hence find the solution of the differential equation

$\sqrt{x^2 + 16} \frac{\mathrm{d}y}{\mathrm{d}x} + y = x\sqrt{x^2 + 16}$	$\sqrt{x^2 + 16} \frac{\mathrm{d}y}{\mathrm{d}x}$	+ <i>y</i> =	$x\sqrt{x^2}$	+1	6
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for which $y = 6$ when $x = 3$.	[6]



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8 (a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^7$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

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 $\cos^7 \theta = a \cos 7\theta + b \cos 5\theta + c \cos 3\theta + d \cos \theta,$

where a , b , c and d are constants to be determined.	[5]
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Let
$$I_n = \int_0^{\frac{1}{4}\pi} \cos^n \theta \, d\theta$$
.

(b) Show that

$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}.$	[4]
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Additional page

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